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The Interaction Between Unemployment Insurance and Human Capital Policies.*

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Abstract

In the presence of an optimally designed unemployment insurance (UI) program, we show that the return to human capital must be equated to the risk-free rate at any constrained efficient allocation. When we specialize to a two period representation of life-cycle choices we show that this requires inducing the agents to accumulate more human capital than they would otherwise do. This policy raises the opportunity costs for those who intend to free ride on the program. We also show that, at the constrained optimum, human capital investments should be driven up to the point where its expected return equals the risk free rate, even though human capital is 'risky' from a private perspective. These results replicate the findings of da Costa and Maestri (2007) in a moral hazard setting that typically characterizes UI programs. When investments in human are not observed by the government, we show that age dependent income taxation may help screen out those who intend to free ride on the UI program. This possibility arises due to the complementarity between human capital choices and labor market attitude which is absent in a UI model that does not incorporate human capital investments. *J.E.L. codes: J65, I28.*

1 Introduction

Economists have long recognized the connection between unemployment episodes and an individual's human capital. On the one hand, unemployment episodes are associated with lower returns to human capital investments, since human capital is of limited use when one is jobless.¹ On the other hand, there is substantial evidence that the more

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¹Human capital may increase productivity in household activities. However, provided that the bulk of the gains from increased human capital takes place in market related activities, the argument remains valid.

educated a person is to the lesser extent she is unemployed — see, for example, [Nickell and Bell \(1997\)](#).

It is apparent from the previous considerations that optimal unemployment insurance — henceforth, UI — and educational policies may have important interactions. It is not so clear, however, how these forces play out to determine optimal policies. On the one hand, human capital typically pays off in the states of nature where the agent is working, i.e., where the marginal utility of consumption is lower. Human capital is, therefore, risky, and we should expect there to be under-investment from a social perspective. On the other hand, more human capital is associated with less frequent and shorter unemployment episodes,² which drives private investments in the other direction. General statements based on under or over-investment are, therefore, bound to be a poor guide for policy.

This paper analyzes the interaction between educational and UI policies, with particular emphasis on the questions of whether the government should influence private choices of human capital. To the best of our knowledge, [Brown and Kaufold \(1988\)](#) were the first to build a theoretical framework to explore the relationship between human capital formation and UI programs. They showed that the presence of a UI program may lead to increased investment in human capital by reducing human capital risk. They also explore various channels through which human capital choices affect the optimal design of such program.

We revisit the work of [Brown and Kaufold \(1988\)](#) within the framework of the new dynamic public finance literature, where optimal policies for the government are derived in a dynamic agency framework. We start with a general setting without specifying the exact nature of the incentive problem and characterize a return condition that must characterize human capital choices in *any* constrained efficient allocation. Our general presentation encompasses, for example, the model of [Atkeson and Lucas \(1995\)](#).

Next we adopt the simple two period structure of their [Brown and Kaufold \(1988\)](#) model, as well as their consideration of non-market activities.³ Also related is [da Costa and Maestri \(2007\)](#) where life-long human capital risk is investigated in a two period self-selection environment. Indeed, one of the purposes of our current work is to verify if their findings apply in a moral hazard setting. This type of comparison follows a series of other studies that investigates the parallels between the two environments. [Golosov et al. \(2003\)](#), for example, show that the inverse Euler equation derived by

²One should, however, be suspicious of endogeneity problems that might be driving the negative correlation between unemployment and education found in the data, since unemployment is not independent of agents' behavior.

³We interpret non-market activities as participation in informal markets, however, our formalization cannot be distinguished from a household production model.

Rogerson (1985) in a dynamic moral hazard model is also present in a dynamic screening problem.⁴ Kocherlakota (2004b) goes on to show that the double deviation result with non-observable savings presented, for example, in Chiappori et al. (1994) for a dynamic moral hazard model also holds in a self-selection setup, while Arnott and Stiglitz (1986) derive supplementary tax rules in a moral hazard problem and compares the results with those in Atkinson and Stiglitz (1976). Finally, Strawczynski (1998) shows that the regressive piecewise linear schedule numerically found by Sheshinski (1989) and Slemrod et al. (1994) in a Mirrlees' setup is reversed in not robust to changing the tax motivation to a social insurance setting. The current paper follows the trend in trying to verify the parallel in the case of human capital policies.

By investigating the problem in an agency framework, we are able to explicitly consider the labor/leisure trade-offs that are not included in their paper. We also emphasize the incentive effects of unemployment insurance in determining the fraction of an agent's productive life in which she is unemployed. At the same time, we are able to derive robust results under weaker assumptions on agents' preferences. The main shortcoming of our approach is that tax systems implicitly derived are more complex and (in some cases) more informationally demanding than those considered in Brown and Kaufold (1988).

Interestingly enough, the model generates a complementarity between education and labor market attitude that endogenously produces the negative correlation between education and unemployment which echoes the empirical evidence that we have mentioned before. The consequence is that the encouragement of human capital formation becomes an important ally for the UI program. Governments that provide insurance networks may often face individuals who claim that they cannot find or keep their jobs when in fact they are not spending enough job-retention and/or job-search effort. It is, therefore, possible for the government to alleviate this problem by raising the opportunity cost of the unemployment spell through higher provision of education.

Notice that the risky or otherwise nature of human capital investment has no bearing here. Human capital acquisition is to be encouraged not because agents under or over-invest in it,⁵ but exactly because the complementarity of search effort and human capital implies that more education signals a 'good' labor market attitude and helps in separating 'unlucky' agents from those who just do not put forth enough effort in participating in formal markets.

⁴This parallel is made explicit by da Costa and Werning (2002)

⁵This type of reasoning plays a role in Brown and Kaufold (1988) model. They apply a result due to Levhari and Weiss (1974) where a multiplicative functional form imposed on the relationship between human capital and intrinsic risk generates increased risk and, as a consequence, under-investment in human capital.

Another finding is that the expected return of human capital investment is equal to the risk free rate, at the optimum. This is a little surprising since human capital is risky. That is, because the UI program must take incentives into account, consumption is higher when an agent is employed than when unemployed. Since human capital investment only pays off in the first case, the private optimal choice implies a risk adjustment for human capital investment. Yet, government intervention drives human capital investment up to the point where its expected return is equal to the risk free rate of return.

Our main results are derived under the assumption that the government controls agents' human capital choices. This assumption is useful in showing the general direction of optimal human capital policy, but may not be entirely feasible in practice. We, thus, consider the possibility that some form of investment is not directly controlled by the government. First, human capital choices. By taking the extreme position that human capital is beyond direct influence by the government, we ask how labor income taxes interact with human capital. What we show is that the interaction between human capital choices and labor market attitude induces an age dependent income tax structure. Young agents are to be taxed and agents at their prime earnings age are to be subsidized at the margin.

The role of savings is carefully discussed in our paper. Savings represent a very important form of self insurance, and whether they are observed or not will determine to a great extent what the government may accomplish in our setting. We first assume that the government can fully control savings. In this case we recover the inverse Euler equation result of [Rogerson \(1985\)](#), and show that labor income should not be taxed at the margin. When savings cannot be controlled by the government the prescriptions regarding the design of labor income taxes used to finance the optimal unemployment insurance program are changed. Nonetheless, the prescriptions for human capital policies are robust to non-observability of savings.⁶

The remainder of paper is organized as follows. Section 2 presents a general setting for which our most general result is proved. Section 3 presents the economy and discusses agent's choices absent government policies. In section 5 we derive our main results. In section 6 we relax the assumption that the Government may choose the agents' human capital, and show how labor income taxation may be helpful in identifying agents that try to free ride in the UI program. In section 7 we discuss the role of non-observed savings that arises when the government tries to implement the

⁶A difficulty that arises in addressing hidden investments is that the interaction between these choices and labor market attitude renders the agents' problem non-convex. We cannot rely on a first order approach, which is in contrast with the dynamic screening problem—e.g. [da Costa and Maestri \(2007\)](#).

second-best inter-temporal transfers. The possibility of these extra hidden choices is accompanied by some technical issues which are handled through a series of results that we present in the appendix. Section 8 concludes.

2 The General Model

Our economy is inhabited by a continuum measure one of identical expected utility maximizing individuals.

Preferences Individuals' preferences are defined over random sequences of consumption and leisure, through a von-Neumann Morgenstern expected utility representation with temporary utility $u(c) - \zeta(\bar{L} - l)$, where c is consumption, l is leisure and \bar{L} is the agent's time endowment. We assume that both functions are smooth with u' , ζ' , ζ'' , $-u'' > 0$, all satisfying usual Inada conditions.

As for considering identical agents, we assume heterogeneity away not for sake of realism but rather for simplicity. The fact that agents are identical means that we disregard the possible interactions between redistributive and insurance motives in government's policy design. Moreover, non-observed heterogeneity and the possibility of self-selection issues may be the very reason for the non-existence of private unemployment insurance, in the first place. Yet, as in Bailey (1978), we remove heterogeneity to focus on the incentive effects associated with the unemployment insurance program.

Also important is the fact that by allowing for a continuum of agents we will be able to focus on a setting without aggregate uncertainty, as discussed below.

Technology Technology is very simple. Output, Y , is the product of an agent's effort, $\bar{L} - l$, and her productivity $w(h)$. Productivity $w(h)$ is a function of an agent's human capital h . That is the rate at which time is transformed in efficiency units, Y , depends on how educated the agent is: $Y = w(h)L$, where $w(h)$ is an agent's productivity. That is, productivity depends on an agent's human capital, h . We assume $w(0) = 1$ and $w' > 0, w'' < 0$: human capital increases an agent's productivity but at decreasing rates.

A linear technology transforms one efficiency unit of labor into one unit of output. We abuse notation slightly by using Y to represent both, and normalize units in such a way that one hour of time produces one efficiency unit of labor in the informal sector.

Finally, to acquire human capital an agent must dedicate some of her time at youth to studying, therefore sacrificing her leisure and/or her first period income. This means that, in the first period, and absent government intervention, $c = Y = \bar{L} - l - h$, where l is leisure and \bar{L} is total time endowment. We take foregone earnings to be the only cost of education. This assumption is simply for notational convenience.

Informational Structure We use θ_t to represent the individual's state in period t . There are two possible states: employed, e , and unemployed. That is, $\theta_t \in \{e, u\}$. An individual's history is represented by $\theta^t = (\theta_1, \dots, \theta_t)$. It is assumed that the probability of one agent finding herself in a given state is a function of some non-observed choice, which we shall not specify in this section.

Although we do not model explicitly the source of informational asymmetry we do assume, that whatever it is it requires utility to differ between employed and unemployed states. Another important implicit assumption is that this 'moral hazard' is not related to an agent's output when employed. In the next section we write a simple two period model for which these assumptions are verified.

We also assume that an agent's probability of being in either state is a function of her actions alone. In particular there are no externalities in this effort. We then use the fact that we have a continuum of identical agents to invoke the law of large numbers argument of [Judd \(1985\)](#) to equate the probability of one being unemployed to the fraction of agents unemployed in each period.

Allocations Allocations in this world are measurable functions $\{c, Y\}$ that may depend on the agent's entire employment history. We use $c(\theta^t)$ to denote the consumption of an agent with history θ^t and $Y(\theta^t)$ as his output. It will also be convenient to define the simplified notation $c^e(\theta^t) \equiv c(\theta^{t-1}, e)$, $Y^e(\theta^t) \equiv Y(\theta^{t-1}, e)$ and $c^u(\theta^t) \equiv c(\theta^{t-1}, u)$. Naturally, unless we explicitly allow for hidden markets, $Y(\theta^{t-1}, u) = 0$.⁷ Define also

$$U^e(\theta^t) \equiv u(c^e(\theta^t)) - \zeta \left(\frac{Y^e(\theta^t)}{w(h)} \right)$$

An allocation $\{c, Y\}$ is feasible if, in every period,

$$\sum_{\theta^{t-1}} \{ \pi(\theta^{t-1}, e) [c^e(\theta^t) - Y^e(\theta^t)] + (1 - \pi(\theta^{t-1}, e)) c^u(\theta^t) \} \leq 0.$$

An allocation $\{c, Y\}$ is incentive compatible if it induces the associated probabilities.

The discription of our economy encompasses the setting of [Atkeson and Lucas \(1995\)](#). Instead of a fixed cost of work, we allow for a variable cost that enriches the space of feasible contracts. That is, our presentation of the economy allows for variable costs of work as an additional instrument—beyond utility promises—for the planner.

⁷In this case, the crucial assumption is that the rate at which time is transformed in efficiency units, Y , in the informal sector, does not depend on how educated the agent is.

2.1 Human Capital in the General Setting

Our goal in this section is to present some properties that any constrained efficient allocation must possess. As of this moment we do not ask who is the principal that is implementing it.

Let, then, $\{c, Y\}$ denote a constrained efficient allocation and consider the following reform. Increase h and reduce $c^e(\theta^t)$ in such a way as to hold $U^e(\theta^t)$ constant at every history (θ^{t-1}, e) and every t .

$$u'(c^e(\theta^t)) dc^e(\theta^t) + \zeta' \left(\frac{Y^e(\theta^t)}{w(h)} \right) \frac{Y^e(\theta^t)}{w(h)^2} w'(h) dh = 0 \quad \forall (\theta^{t-1}, e) \forall t$$

In order to hold utility constant in the first period, as well, one must change c_0 as well.

$$u'(c_0) dc_0 - \zeta'(h) dh = 0$$

Note that we are considering the case in which Y is 0 at youth.

The important thing to realize is that, because utility is held constant across all histories, no incentives are affected with this reform. The total cost of such reform is

$$dc_0 + \sum_t \sum_{\theta^{t-1}} \pi(\theta^{t-1}, e) dc^e(\theta^t).$$

If the initial allocation is optimal, the reform's cost must be zero, that is,

$$\frac{\zeta'(h)}{u'(c_0)} dh + \sum_t \sum_{\theta^{t-1}} \pi(\theta^{t-1}, e) \zeta' \left(\frac{Y^e(\theta^t)}{w(h)} \right) \frac{Y^e(\theta^t)}{w(h)^2} \frac{w'(h)}{u'(c^e(\theta^t))} dh = 0$$

or

$$\frac{\zeta'(h)}{u'(c_0)} = \sum_t \sum_{\theta^{t-1}} \pi(\theta^{t-1}, e) \frac{1}{u'(c^e(\theta^t))} \zeta' \left(\frac{Y^e(\theta^t)}{w(h)} \right) \frac{Y^e(\theta^t)}{w(h)^2} w'(h)$$

Next, note that

$$\frac{\zeta'(h)}{u'(c_0)} = \zeta' \left(\frac{Y^e(\theta^t)}{w(h)} \right) \frac{1}{w(h) u'(c^e(\theta^t))} = 1,$$

which then implies

$$1 = \sum_t \sum_{\theta^{t-1}} \pi(\theta^{t-1}, e) \frac{Y^e(\theta^t)}{w(h)} w'(h).$$

We express the result above in the following proposition.

Proposition 1. *At any (constrained) efficient allocation, the expected return to human capital is equal to one.*

We may now contrast this with the optimal choice of h by the agent,

$$\zeta'(h) = \sum_t \sum_{\theta^{t-1}} \pi(\theta^{t-1}, e) \zeta' \left(\frac{Y^e(\theta^t)}{w(h)} \right) \frac{Y^e(\theta^t)}{w(h)^2} w'(h),$$

or

$$1 = \sum_t \sum_{\theta^{t-1}} \pi(\theta^{t-1}, e) \frac{u'(c^e(\theta^t))}{u'(c_0)} \frac{Y^e(\theta^t)}{w(h)} w'(h).$$

The question is how the two conditions compare. If allowed to freely choose, or absent any distortions at this margin, individuals equate the marginal cost of their investment to the risk adjusted return to this investments. Imagine that, at youth, an agent must decide whether to go an extra year to school or whether she should quit school, work and save for the future. The risk adjusted returns on both types of invesments must be equated.

Thus, despite the fact that there is no full insurance — which makes human capital investment risky —, government policies induce agents to increase investment in human capital up to the point where its expected return is equal to one. If individuals had access to a risk-free asset, then they would equate the risk adjusted return on education with the fixed return of this asset. What we shall recall next is that this is not the case. Optimal allocations imply the impossibility of free access to such assets.⁸ The proof that follows is the one in [Rogerson \(1985\)](#). We repeat it for completeness.

Indeed, starting with allocation $\{h, c, Y\}$ define a new allocation $\{\hat{h}, \hat{c}, \hat{y}\}$ such that $\hat{h} = h$, $\hat{y} = Y$ and $\hat{c}(\theta^s) = c(\theta^s)$ for $s \neq t-1$, $s \neq t$. For all θ^{t-1} change consumption by $\hat{c}(\theta^{t-1}) = c(\theta^{t-1}) + \delta(\theta^{t-1})$ and compensate in the next period by changing consumption by $\hat{c}(\theta^t) = c(\theta^t) + \varepsilon(\theta^t)$ for all θ^t following θ^{t-1} . That is for each θ^{t-1} and all its continuations θ^t , $\delta(\theta^{t-1})$ and $\varepsilon(\theta^t)$ are implicitly defined by

$$u(c(\theta^{t-1}) + \delta(\theta^{t-1})) + u(c(\theta^t) + \varepsilon(\theta^t)) = 0.$$

For small ε ,

$$\varepsilon(\theta^t) = -\frac{u'(c(\theta^{t-1}))}{u'(c^e(\theta^t))} \delta(\theta^{t-1})$$

⁸More precisely, agents can participate on those markets provided that tax systems are such that the after tax return of these assets does not affect incentives. E.g., [Kocherlakota \(2004b\)](#).

The difference in cost between this and the former allocation is

$$\delta(\theta^{t-1}) + \pi(\theta^{t-1}, e) \varepsilon(\theta^{t-1}, e) + \pi(\theta^{t-1}, u) \varepsilon(\theta^{t-1}, u) = 0$$

or

$$\varepsilon(\theta^t) = -\frac{u'(c(\theta^{t-1}))}{u'(c(\theta^t))} \delta(\theta^{t-1}).$$

The constrained efficient allocation must minimize costs, which implies that

$$\frac{1}{u'(c(\theta^{t-1}))} = \left[\frac{\pi(\theta^{t-1}, e)}{u'(c(\theta^{t-1}, e))} + \frac{\pi(\theta^{t-1}, u)}{u'(c(\theta^{t-1}, u))} \right].$$

The inverse Euler equation holds, which is nothing but Rogerson's result. For our purposes, the important thing is that because there is a wedge between marginal utility of consumption and expected marginal utility of consumption (multiplied by the risk-free return to savings) one cannot guarantee that agents would want to substitute savings for investment in human capital.

3 The Two Period Economy

We now specialize to a two period economy with an atom of identical agents. The two period assumption follows [Brown and Kaufold \(1988\)](#) and [Bailey \(1978\)](#), to name a few,⁹ and is mainly due to our emphasis in the interaction between education and unemployment. When compared to the length of unemployment spells, educational choices are usually long term and mostly done early in life. [Brown and Kaufold \(1988\)](#) emphasize their effect on later choices regarding work since education changes the relative payoff of employment *vis à vis* unemployment. We focus on the converse: labor market attitude influence educational choices.

Preferences Preferences over sequences of consumption and leisure are as in section 2. Beyond labor supply, however, we include another dimension of effort, not belonging to the description of temporary utility, related to the struggle to remain in the formal markets. The fraction of time of an agent's adult life that she spends unemployed is a function of both the (per period) probability of her losing a job when employed and the probability that she gets a new job when unemployed. We capture both transition probabilities with a single variable $p \in [0, 1 - \varepsilon]$. We associate it with the agent's attitude toward work: be it her willingness to conform to different rules or environments, encompassed in the general label of job-retention effort, be it her search

⁹[Acemoglu and Shimer \(2001\)](#) is a one period representation, which can be derived from a fully dynamic model.

effort whenever unemployed. We assume p to be strictly less than one to avoid the (unrealistic) policy of extreme punishments for any agent who is ever unemployed. We take the choice of p to be a life-long choice, that produces a utility cost which we represent with $\varphi(\cdot)$: an increasing, convex, continuously differentiable function.¹⁰ In section 5.1 we consider the case where, instead of a utility loss, the cost of remaining in the formal markets is earnings loss, which we interpret as a reduced form of a search model.

With all this in mind, we write an agent's life-time utility as

$$u(c) - \zeta(\bar{L} - l) + \mathbb{E}[u(c) - \zeta(\bar{L} - l)] - \varphi(p), \quad (1)$$

where the expectation operator in (1) is with respect to the probability p .

Technology We consider in this section that the economy has two sectors: a formal sector and an informal sector. Each sector produces the single consumption good with a linear technology that transforms one efficiency unit of labor into one unit of output. The crucial assumption is that the rate at which time is transformed in efficiency units, Y , in the informal sector, does not depend on how educated the agent is. In contrast, in the formal sector, $Y = w(h)L$, where $w(h)$ is an agent's productivity.

It is natural to think that productivity is higher in the formal sector. Absent this, a formal sector would not exist in equilibrium. Our assumptions imply that productivity is higher for all levels of human capital. Underlying it is the idea that the technology that is available in the informal sector can be adopted by the formal sector, but not necessarily the other way around.

We rule out the participation in the formal sector in the first period: we are concerned with unemployment at an agent's prime age. We shall allow for this possibility in section 6 to discuss how income taxes may help induce optimal behavior when investment in human capital is not fully observable.

Because we have ruled out aggregate risk, p will be associated not only with the fraction of the agent's adult life that she is employed but also with the unemployment rate of the economy. In fact, the steady state ratio unemployment/employment is equal to the ratio of the probability of transition from employment to unemployment to the probability of transition from unemployment to employment which are both captured in our model with the single parameter p .¹¹

¹⁰Two important restrictions are imposed on p . First, it increases labor-market participation without increasing productivity while employed. Second, it cannot be altered later in life. While the first characteristic brings p closer to search effort, the second brings it closer to another dimension of human capital. We shall push the first interpretation a bit further in section 5.1 but not the second. Nonetheless, one should note that the model in section 2 did not rule out this possibility.

¹¹In a steady state, these transition probabilities are also the inverse of the length of employment

4 First Best and Autarchy Allocations

In this section we evaluate first best and equilibrium allocations in a world where the only form of reducing risk is self-insurance through savings, which we shall refer to as autarky.

Autarchy Consider the case in which there is no unemployment insurance. Letting s denote savings, we may write the agent's problem as

$$\max_{p, Y, h, s} \{u(Y - s) - \zeta(h + Y) + pV^e(h, s) + (1 - p)V^u(s) - \varphi(p)\},$$

where

$$V^u(s) = \max_{Y^u} \{u(Y^u + s) - \zeta(Y^u)\}, \quad (2)$$

and

$$V^e(h, s) = \max_{Y^e} \left\{ u(Y^e + s) - \zeta\left(\frac{Y^e}{w(h)}\right) \right\}, \quad (3)$$

where we use Y to denote output in the first period, Y^e to denote output in the second period while employed in the formal markets, and Y^u to denote output in the second period when working in the informal markets.

In what follows, to make the problem interesting we shall adopt the following assumption.

Assumption A: *The optimum for the autarchy problem entails $p^* > 0$.*

Notice that the agent's problem need not be convex, due to the interplay between p , s and h . Nonetheless, provided that the solution is interior, the following first order conditions are necessary:

$$u'(Y + s) = p\partial_s V^e(h, s) + (1 - p)\partial_s V^u(s), \quad (4)$$

$$V^e(h, s) - V^u(s) = \varphi'(p), \quad (5)$$

$$u'(Y + s) = \zeta'(h + Y), \quad (6)$$

and

$$\zeta'(h + Y) = p\partial_h V^e(h, s). \quad (7)$$

Finally, applying the envelope theorem to (3) we may rewrite (7) as

$$\zeta'(h + Y) = p\partial_h V^e(h, s) = p\zeta'\left(\frac{Y^e}{w(h)}\right) \frac{Y^e}{w(h)^2} w'(h). \quad (8)$$

and unemployment spells, respectively.

First Best Let c , c^e and c^u denote, respectively, consumption in the first period, in the second period if employed and in the second period if unemployed and consider the first best allocations, in which the possibility of transfers from employed to unemployed agents is given to a social planner.

$$\max \left\{ u(c) - \zeta(h + Y) + p \left[u(c^e) - \zeta \left(\frac{Y^e}{w(h)} \right) \right] + (1 - p) [u(c^u) - \zeta(Y^u)] - \varphi(p) \right\},$$

subject to

$$c + pc^e + (1 - p)c^u = Y + pY^e + (1 - p)Y^u. \quad [\mu]$$

Once again, one should beware with the fact that the problem need not be convex, thus, first order conditions are only necessary (once again, assuming that the solution is interior). They are,

$$u'(c) = u'(c^e) = u'(c^u) = \mu, \quad (9)$$

$$\zeta' \left(\frac{Y^e}{w(h)} \right) \frac{1}{w(h)} = \zeta'(h + Y) = \mu, \quad (10)$$

$$u(c^e) - \zeta \left(\frac{Y^e}{w(h)} \right) - [u(c^u) - \zeta(Y^u)] + \lambda [Y^e - c^e - Y^u + c^u] = \varphi'(p), \quad (11)$$

and

$$\zeta'(h + Y) = p\zeta' \left(\frac{Y^e}{w(h)} \right) \frac{Y^e}{w(h)^2} w'(h). \quad (12)$$

The fact that (8) and (12) are identical implies that, *conditional on identical labor supply choices*, educational choice are identical in the first best and in autarchy. However, labor supply will not be the same,¹² and neither will human capital choices be. Yet, simple inspection of first order conditions does not allow us to tell whether the government should distort human capital choices absent other policies.

5 Optimal Policy

We set up the government's program as a mechanism design problem and derive the optimal allocations leaving the policy instruments in the background. Our model is, with regards to human capital choices, close to [Hamilton \(1987\)](#) where the level of human capital chosen by the individuals is compared to the level which the government chooses when it has the power to do so, or when the instruments necessary to induce

¹²Notice that $u'(Y^e + s) < u'(y - s)$ in the autarchy example. Which implies, when one considers the first order conditions of (3) and (2), that the equality in (7) cannot hold in autarchy. As a consequence, the condition $1 = pL^e w'(h)$ found by combining (7) with (12) is not valid in the autarchy case, where the returns to investment in human capital must be 'risk adjusted'.

such choices. In section 5.1, we briefly discuss the implementation of these allocations.

Back to the mechanism design problem, we assume that the government controls are: *i*) a transfer δ to the first-period of each agent's life; *ii*) the unemployment insurance policy, which takes the form of a transfer ω to the agent in case she loses her job, *iii*) labor supply and consumption choices Y^e , y^e while employed; *iv*) the 'labor market attitude', p , and; *v*) the agents human capital, h .

The choice of p is made under the restriction that the agent will only choose the level of p prescribed by the government if she finds in her best interest to do so. I.e., p must satisfy the associated incentive compatibility constraint,

$$p \in \arg \max \left\{ p \left[u(y^e) - \zeta \left(\frac{Y^e}{w(h)} \right) \right] + (1-p) V^u(\omega) - \varphi(p) \right\}. \quad (13)$$

The government cannot observe the amount of work the agent supplies in the hidden economy, both when young, Y , and when adult, Y^u . Nonetheless, because the government controls h and s , the problem of the agent is convex in the remaining choice variables, which means that the first order approach may be applied, and (13) replaced with

$$u(y^e) - \zeta \left(\frac{Y^e}{w(h)} \right) - V^u(\omega) = \varphi'(p). \quad (14)$$

The government, thus, solves the following problem,

$$\max_{h,p,y^e,Y^e,\omega,\delta} \left\{ V(h,\delta) + p \left(u(y^e) - \zeta \left(\frac{Y^e}{w(h)} \right) \right) + (1-p) V^u(\omega) - \varphi(p) \right\}, \quad (15)$$

where

$$V(h,\delta) \equiv \max_Y \{ u(Y + \delta) - \zeta(h + Y) \},$$

and

$$V^u(\omega) \equiv \max_{Y^u} \{ u(Y^u + \omega) - \zeta(Y^u) \},$$

subject to the resource constraint,

$$p(Y^e - y^e) \geq (1-p)\omega + \delta \quad [\mu]$$

and the incentive constraint (14) to which we associate the Lagrange multiplier λ .

Walras' identity allows us to leave the government's budget constraint in the background; if the resource constraint is met, so is the government budget constraint. The underlying assumption is that the government need only balance its budget intertemporally, which implicitly allows for some external markets for borrowing and saving. As in Brown and Kaufold (1988), our model is a better approximation of an open economy.

Since our main concern here is the educational policy, we differentiate the associated Lagrangian (the multipliers are as shown above) by h to get the first order condition,

$$(p + \lambda) \zeta' \left(\frac{Y^e}{w(h)} \right) \frac{Y^e}{w(h)^2} w'(h) = \zeta' (h + Y). \quad (16)$$

Assume for now that the solution to the agent's optimization problem is interior (the corner solution $h = 0$ is obvious). Then, if the government does not intervene in human capital formation, the agent's optimal choice of education would be characterized by the first order condition

$$p \zeta' \left(\frac{Y^e}{w(h)} \right) \frac{Y^e}{w(h)^2} w'(h) = \zeta' (h + Y). \quad (17)$$

By comparing (16) with (17) it is apparent that the government should distort the agent's choice thus creating a wedge between private marginal costs and private marginal benefits of education.

Proposition 2. *At the (constrained) optimum, the government induces agents to choose a level of human capital, h , higher than what an agent would choose if the government did not intervene in this choice.*

The proof is by comparing (16) and (17), using the convexity and monotonicity of $\zeta(\cdot)$ and the concavity and monotonicity of $w(\cdot)$.

The optimal policy comprises the government inducing agents to increase human capital investment, as formally stated in the following proposition. By inducing agents to over-accumulate human capital, the government increases the opportunity cost of being unemployed, thus alleviating the moral hazard problem that undermines the risk sharing possibilities of the UI program.

Next, notice that the first order conditions with respect to y^e and h can be manipulated to obtain the following alternative representation of the wedge,

$$\frac{1}{1 + \lambda/p} = p \frac{u'(y^e)}{u'(c)} L^e w'(h). \quad (18)$$

The private marginal benefit of education which appears in the right hand side of (18) displays a state-price deflator that adjusts for risk involved in human capital investment. If a moral hazard problem were not present in this setting we would have full insurance and the optimal level of human capital would be found by equalizing its expected return to that of the risk free asset. Here, however, some risk is born by the agent making human capital 'risky'.

Risk adjustment will reduce, for every h , the right hand side of (18), as compared to its expected return. The fact the left hand side of (18) is less than one, however, precludes its use for comparing the expected return of human capital with that of the risk free asset. The reason why such comparison is important is because when the expected returns on the two assets are equal, i.e., $1 = pL^e w'(h)$, production is taking place at the technological frontier in the sense that there is no way for aggregate consumption to be increased in both periods, at current choices of leisure — or, alternatively, for leisure to be increased at the current level of consumption.

This is where the possibility of inter-temporal transfers, as represented by δ , becomes important. Differentiating the government's problem with respect to δ and using the envelope theorem we have $u'(c) = \mu$. Thus, $u'(y^e)/u'(c) = (1 + \lambda/p)^{-1}$, which recovers the result in that investment in human capital is driven up to the point where its expected return is equal to one.

This result is simply a corollary of Proposition . Our derivation in this section simply highlights the importance of the planner's control over an agent's consumption.

It is worth mentioning the fact that the marginal tax on labor income is zero.¹³ This is immediate from the first order conditions with respect to y^e and Y^e , respectively,

$$(p + \lambda) u'(y^e) = p\mu, \text{ and, } (p + \lambda) \zeta' \left(\frac{Y^e}{w(h)} \right) \frac{1}{w(h)} = p\mu.$$

There is a sense in which the result is to be expected since agents are homogeneous and lump-sum taxes, feasible. However, labor income taxation may still help if labor income is used by the agents in connection with savings or human capital choices to allow for free riding in the unemployment insurance program. This is exactly the case in sections 6 and 7. As we shall see, no-distortions at the margin ceases to be optimal.

5.1 Discussions and a Caveat

The Role of Informal Markets. The informal sector adds an important, often neglected dimension to labor market description. This is particularly true for under-developed economies, but the point is more general. Nonetheless, the inclusion of an informal market as part of the description of the economy may lead one to wonder how important this is for the results we obtain — and, consequently, how relevant this may be for developed economies. The answer is that all the results remain valid if we remove the informal sector.

¹³Note also that, if labor income in the first period could be taxed, the optimal marginal tax rate would also be zero. A slight change in the model allows for an immediate proof of this result.

To see this just notice that when problem (15) is replaced with

$$\max \left\{ u(y) - \zeta(Y + h) + p \left(u(y^e) - \zeta \left(\frac{Y^e}{w(h)} \right) \right) + (1 - p) [u(\omega) - \zeta(0)] - \varphi(p) \right\},$$

where maximization is with respect to y, Y, h, p, y^e, Y^e and ω , subject to the appropriately modified constraints, the exact same expressions, (16) and (18), obtain. Hence, the result is not dependent on the existence of an informal market but rather robust to its existence!

Search. We have modeled the cost of ‘having the right attitude’ as an additive utility cost, $\varphi(p)$. In many studies in which one concentrates on the transition from unemployment to employment (taking the employment tenure as given) search is modeled as a sequence of unobserved (by the planner) wage offers that the agent may or may not accept. In this case, higher effort means accepting lower wages, which means that, in a reduced form, we should write the agent’s life-time utility as

$$u(y) - \zeta(Y + h) + p \left(u(c^e) - \zeta \left(\frac{Y^e + \varphi(p)}{w(h)} \right) \right) + (1 - p)V^u.$$

where $Y^e = \hat{Y}^e - \varphi(p)$ is observed but not \hat{Y}^e and $\varphi(p)$ in isolation.¹⁴ In this case, $c^e = Y^e - T(Y^e) = \hat{Y}^e - \varphi(p) - T(\hat{Y}^e - \varphi(p))$.

The result regarding human capital formation not only survives this change but is actually strengthened by the fact that education also reduces the marginal cost of search. It is also easy to verify that the inverse Euler equation still characterizes the inter-temporal distribution of consumption.

The only major change is with regards to the zero marginal tax on labor income which is now replaced by a negative marginal income tax.

Implementation. So far, we have not said a word about how these optimal policies may be implemented. That is, what do we mean by having the government choose h and s ? Even though the same type of question applies to Y^e , we are so used to [Mirrlees’s \(1971\)](#) approach that we sometimes fail to recognize that the two problems are of the same nature.

We shall not discuss in detail the issue of implementation, not because we do not think that they are interesting but because the type of problems that arise here are well understood, and careful discussions are found in the literature — see [Chiappori et al. \(1994\)](#) for the problem of double deviation in a moral hazard context, and [da Costa and Maestri \(2007\)](#) for tax systems that allow the government to effectively control savings

¹⁴This cost is due to a mismatching between the job and the agents, an assumption adopted to prevent firms from having positive profits.

and human capital investment. We shall, however, point out to the fact that linear, non-stochastic taxes on both forms of investment need not substitute for compulsory choices. That is, a simple subsidy on h and a simple tax on s may not implement the optimal choices because of the double-deviation problem discussed in [Chiappori et al. \(1994\)](#). Some form of non-linear or state-dependent tax may be necessary, in this case.

Inter-temporal transfers. An important caveat associated with our main result concerns the role of inter-temporal transfers. When arguing that the assumption that the government controls savings is not a very restrictive one, we used the fact that credit markets may not be generous enough to allow for negative savings when such long horizons are considered. With inter-temporal transfers, however, the optimal policy is characterized by an inverse Euler equation that implies¹⁵

$$u'(Y + \delta) < pu'(y^e) + (1 - p)u'(Y^u + \omega).$$

The consequence is that the non-negativity restriction on savings ceases to be relevant and the potential non-observability of savings has important consequences for policy design.

There are two possible reactions to this issue. First we may argue that non-observability is not important so that, in practice, the government controls savings.¹⁶ The second possibility is to recognize that non-observability is important and optimal policies should take this into account.¹⁷

6 Non-observed Human Capital

So far we have been taking for granted the capacity of government to control agents' human capital choices. This may not, however, be a very good approximation of real world institutions. For example, there is an important dimension of investment that is not observed which is the effort placed on learning activities.

In this section we consider the case in which investment in human capital is not observed or controlled by the government. Of course, the assumption that human capital is completely non-observed is also far-fetched. However, by taking an extreme

¹⁵This is but a restatement of [Rogerson \(1985\)](#) result easily derived in our model by combining the first order conditions with respect to δ , ω and y^e .

¹⁶E.g., [Shavel and Weiss \(1979\)](#), [Hopenhayn and Nicolini \(1997\)](#), [Wang and Williamson \(1996\)](#), [Zhao \(1999\)](#).

¹⁷At the limit, perfect capital markets, idiosyncratic shocks and very low discount, leads to a great deal of self-insurance. In practice, however, unemployment *does* matter even when savings are used to smooth consumption. Without a UI program, [Gruber \(1997\)](#) estimates that the drop in consumption due to unemployment would be three times as large as the average fall in the presence of current UI programs in the US (6.8 percent with the program and 22.2 without it).

opposite position from the one we have adopted so far, we highlight some of the issues that may arise in practice and hint to some alternatives that can play a role in the design of government policies.

We recall the agent's program

$$\max_{p,h} \{u(c) - \zeta(h + Y) + pV^e(h) + (1 - p)V^u - \varphi(p)\}, \quad (19)$$

where

$$V^u(\omega) \equiv \max \{u(Y^u + \omega) - \zeta(Y^u)\}, \quad (20)$$

and

$$V^e(h) \equiv u(c^e) - \zeta\left(\frac{Y^e}{w(h)}\right), \quad (21)$$

We shall also depart from the assumption that work in first period takes place in informal markets to investigate optimal income taxation of youngsters.

We have seen that when free to choose both human capital and non-markets skills, the agent's problem is potentially non-convex. This makes the use of a first order approach unreliable: since the sets generated by (28) and the sets generated by the agents' first order conditions may differ, one cannot substitute the latter for the former when solving the government's problem, in general. That is, because the agent's problem is not convex we will not be able to use the first order conditions associated with (19) to derive optimal policies.

There are some alternatives for dealing with the issue.¹⁸ [Werning \(2002\)](#) restricts preferences to a class where the first order approach is guaranteed to work. [Abrahám and Pavoni \(2005\)](#) solve the model assuming that the approach works and check whether, for the specific parametrization they have chosen, the first order conditions characterize a maximum at the optimal solution. Both models are substantially more complex than ours since these authors work with fully dynamic problems which require the use of recursive methods. The payoff we obtain from working in a simplified environment is that we are able to adopt a procedure that does not rely on a specific functional form and/or parametrization of the problem.

Our procedure consists in discretizing the effort space by redefining the domain of p as the finite set $\mathbb{P} \equiv \{p_0, p_1, \dots, p_N\}$ with $p_0 = 0 < p_1 < \dots < p_N = 1$. Next, we characterize the optimal deviation strategies and verify which ones bind at the optimum and what this implies for the design of optimal policies. This procedure mimics, in some sense, a numerical approach with an important advantage: all results derived herein

¹⁸The issue also arises when savings, instead of human capital choices are not observed, as we shall see in section 7.

are independent of any specific parametrization or functional forms beyond the ones we have been using all along!

Toward our goal, we define

$$h(p) \equiv \arg \max_h \{u(c) - \zeta(h + Y) + pV^e(h) + (1 - p)V^u - \varphi(p)\} \quad (22)$$

and consider the agent's optimization problem with respect to p ,

$$p \in \arg \max_{\tilde{p} \in \mathbb{P}} \{u(c) - \zeta(h(\tilde{p}) + Y) + \tilde{p}V^e(h(\tilde{p})) + (1 - \tilde{p})V^u(\omega) - \varphi(\tilde{p})\}. \quad (23)$$

Next, we consider that the government solves the optimization problem,

$$\max_{p, c, Y, Y^e, \omega} \{u(c) - \zeta(h(p) + Y) + pV^e(h(p)) + (1 - p)V^u(\omega) - \varphi(p)\} \quad (24)$$

subject to (23) and

$$Y - c + p(Y^e - c^e) - (1 - p)\omega \geq 0 \quad (25)$$

Labor Income Taxes In the appendix we show that, although unable to directly pick an optimal h for the agent, the government will optimally exploit the interaction between labor market attitude, human capital choices and labor supply to create incentives for agents not to free ride in the UI program.

We start with the following proposition.

Proposition 3. *When human capital investments are not observed by the government, labor income of young agents should be taxed at the margin.*

In our model, labor market activities compete with education when agents are young, i.e., their cost structure is one of perfect substitutability. It is then possible to show that someone who has a bad labor market attitude (low p) under-invests in human capital thus having a lower disutility of work early in life. A small tax on first period labor income that has no first order welfare impact along the equilibrium path, hurts these deviant agents and opens up space for more insurance.

Notice that our argument depends on the fact that all relevant deviant strategies entail a lower choice of p , which is proven in lemma 3.

As for agents at their prime earnings age, we have the following result.

Proposition 4. *When human capital investments are not observed by the government, labor income of agents at their prime age should be subsidized at the margin.*

To understand the intuition behind this result note that a small subsidy on employed agents who have a good market oriented attitude has no first order welfare effects. This is not true for agents who are trying to free ride in the UI program. The rationale is subtle and comes through the interaction between human capital choices and labor market attitude. Agents with low human capital are hurt by the very subsidies that have no first order impact on agents with higher human capital. Because low human capital signals bad labor market attitude the result follows.

The Inverse Euler Equation Before moving on to the problem of hidden savings, we should mention that, as before, an inverse Euler equation *à la* Rogerson (1985) characterizes intertemporal distortions,

$$\left[\frac{1-p}{u'(c^u)} + \frac{p}{u'(c^e)} \right]^{-1} = u'(c).$$

This is in contrast with Grochulski and Piskorski (2005), where hidden human capital investments leads to the violation of this condition. In Grochulski and Piskorski (2005) non-observability of investment in human capital formation is due to the fact that they cannot be distinguished from the consumption aspect of education. The key to the difference is, therefore, that consumption itself becomes a non-observable variable in their model, but not in ours.

7 Hidden Savings

There is a growing literature dealing with the effects of hidden savings on the design of unemployment insurance programs — e.g., Kocherlakota (2004a), Werning (2002), Abrahám and Pavoni (2005). Our two period framework does not allow us to discuss how savings affect the pattern of transfers along an unemployment spell.¹⁹ Nevertheless, we share with this literature, the concern with how incentives are affected by savings and how this feeds back to the design of government policies.

We first define the indirect utility functions of an unemployed agent,

$$V^u(s, \omega) \equiv \max_{Y^u} \{u(Y^u + \omega + s) - \zeta(Y^u)\},$$

¹⁹We are concerned with savings from youth, when educational choices are made, to adulthood. This is a different issue from that of how savings by one who is a labor market participant affects the design of unemployment insurance policies. (e.g., Werning (2002), Kocherlakota (2004a)).

and that of an employed agent,

$$V(s, \delta, h) \equiv \max_Y u(Y - s + \delta) - \zeta(Y + h).$$

With these definitions, the government's program is

$$\max \left\{ V(s, \delta, h) + p \left[u(y^e + s) - \zeta \left(\frac{Y^e}{w(h)} \right) \right] + (1 - p)V^u(Y, s, \omega) - \varphi(p) \right\}, \quad (26)$$

subject to the resource constraint,

$$p(Y^e - y^e) \geq (1 - p)\omega + \delta, \quad (27)$$

and to the incentive constraint,

$$(p, s) \in \arg \max_{(\hat{p}, \hat{s})} \left\{ V(\hat{s}, \delta, h) + \hat{p} \left[u(y^e + \hat{s}) - \zeta \left(\frac{Y^e}{w(h)} \right) \right] + (1 - \hat{p})V^u(\hat{s}, \omega) - \varphi(\hat{p}) \right\}. \quad (28)$$

To handle the non-covexity of the agent's problem, we shall discretize the space of probability choices— $p \in \mathbb{P} \equiv \{p_0, p_1, \dots, p_N\}$ with $p_i > p_{i-1}$ $i = 1, \dots, N$ and $p_0 = 0$, $p_N = 1$ —and proceed as in section 6.

First, define $W(\delta, h, y^e, Y^e, \omega, p) \equiv$

$$\max_{\hat{s} \in \mathbb{R}_+} \left\{ V(\hat{s}, \delta, h) + p \left[u(y^e + \hat{s}) - \zeta \left(\frac{Y^e}{w(h)} \right) \right] + (1 - p)V^u(\hat{s}, \omega) - \varphi(p) \right\}. \quad (29)$$

the maximum utility the worker attains by optimally choosing her savings, conditional on a given p . The restriction that $\hat{s} \in \mathbb{R}_+$ is due to the credit constraint.

Next, we define for the government a *relaxed* program,

$$\begin{aligned} \max_{\delta, h, y^e, Y^e, \omega, \hat{p}} & W(\delta, h, y^e, Y^e, \omega, \hat{p}) + \mu[\hat{p}(Y^e - y^e) - (1 - \hat{p})\omega - \delta] \\ & + \sum_{p < \hat{p}} \lambda(p)[W(\delta, h, y^e, Y^e, \omega, \hat{p}) - W(\delta, h, y^e, Y^e, \omega, p)] \end{aligned} \quad (30)$$

where, instead of considering the entire set of incentive compatibility constraints, we only take into account the downward ones: those which guarantee that the agent does not choose a lower level of effort than the optimal, p^* .

Because the government faces fewer constraints, the solution to (30) is not inferior to the solution to the government's problem (26) when constraint (28) is considered.

What we show in the appendix is that if $(\delta^*, h^*, y_e^*, Y_e^*, \omega^*, p^*)$ solves (30) then, at this solution, there is no strategy with $p > \hat{p}$ (and associated optimal choices) that yields higher expected utility for the agent. Therefore, $(\delta^*, h^*, y_e^*, Y_e^*, \omega^*, p^*)$ solves government's problem (26) subject to (27) and (28).

For our purposes, the fact that only downward constraints bind will be of paramount importance in identifying the relevant deviating strategies. Along these lines, the next two lemmas, proved in the appendix, are stated here to facilitate the intuition regarding some of the results that follow.

Lemma 1. *In all strategies that contemplate a lower level of effort than the optimal, $p < p^*$, the agent saves at least as much as when she makes the optimal effort, p^* .*

Lemma 2. *In all strategies that contemplate a lower level of effort than the optimal, $p < p^*$, the agent supplies at least as much labor in the first period.*

Underlying these results is the fact that, if the relevant deviating strategies are the ones that contemplate lower effort, savings are complementary to deviant behavior. Agents who do not make enough effort to remain in the formal markets have a higher expected marginal utility of consumption, when compared to agents who choose the optimal effort, p^* . By the same token, higher savings increase first period marginal utility of income thus implying a higher propensity to work in the first period — lemma 2. That is, agents who anticipate being unemployed more often or for longer periods work more on informal activities from very early in their lives, and hold more wealth.²⁰

Educational Policy. The qualitative results regarding the educational policy are not altered by the possibility of hidden savings. It is still optimal for the government to encourage the acquisition of human capital. To show this, we write the first order necessary condition with respect to h ,

$$\begin{aligned} -\zeta'(h + Y^*) + p^* \zeta'(L^e) L^e \frac{w'(h)}{w(h)} + \sum_{p < p^*} \lambda(p) [\zeta'(h + Y(p)) - \zeta'(h + Y^*)] \\ + \zeta'(L^e) L^e \frac{w'(h)}{w(h)} \sum_{p < p^*} \lambda(p) [p^* - p] = 0, \end{aligned} \quad (31)$$

noting that we have applied the envelope theorem in (29) to find the partial derivative of W with respect to h .

²⁰This may not be robust to the existence of multiple unemployment episodes.

It is apparent from lemma 2 that the third and the forth terms in (31) are positive. Therefore,

$$p^* \zeta'(L^e) L^e \frac{w'(h)}{w(h)} < \zeta'(h + Y^*). \quad (32)$$

The inequality above shows that the optimal policy requires the creation of a wedge between optimal private costs and benefits of education, which yields the next proposition.

Proposition 5. *At the optimum, $h^* > h^o$, where $h^o \equiv \arg \max_h W(\delta, h, y^e, Y^e, \omega, p^*)$.*

The proof uses convexity of $\zeta(\cdot)$, concavity of $w(\cdot)$ and inequality (32). The government must induce a choice of h that is higher than the private optimum. The rationale is once again that, by forcing agents to get more education, the government raises the costs of free riding on the unemployment benefit program.

Labor income taxation and UI. Next, we investigate the consequences of hidden savings for optimal labor income taxes and unemployment benefits. We begin by taking the first order conditions with respect to y^e , ω and δ , respectively,

$$\mu = u'(c_e^*) + \sum_{p < p^*} \lambda(p) \left[u'(c_e^*) - \frac{p}{p^*} u'(c^e(p)) \right],$$

$$\mu = u'(c_u^*) + \sum_{p < p^*} \lambda(p) \left[u'(c_u^*) - \frac{1-p}{1-p^*} u'(c^u(p)) \right],$$

and

$$\mu = u'(c_0^*) + \sum_{p < p^*} \lambda(p) [u'(c_0^*) - u'(c_0(p))].$$

Combining the three first order conditions above, we get

$$\begin{aligned} u'(c_0^*) - p^* u'(c_e^*) - (1-p^*) u'(c_u^*) = \\ \frac{\sum_{p < p^*} \lambda(p) [u'(c_0(p)) - p u'(c^e(p)) - (1-p) u'(c^u(p))]}{1 + \sum_{p < p^*} \lambda(p)}. \end{aligned}$$

We know from the first order condition of the agent's savings problem that $u'(c_0^*) = \mathbb{E} u'(c_1^*)$, (expectation is with respect to probability p^*). If, we consider lemmas 1 and 2 and the expression above, along with this latter equality, then, $p < p^* \Rightarrow s^* < s(p)$. This being the case, the first order conditions with respect to Y^{e*} and w^{e*} yield

$$\begin{aligned} \frac{\zeta'(L^e)}{w(h)} \left\{ p^* + \sum_{p < p^*} \lambda(p) [p^* - p] \right\} \\ = p^* u'(c_e^*) + \sum_{p < p^*} \lambda(p) [p^* u'(c_e^*) - p u'(c^e(p))], \end{aligned}$$

which finally implies

$$\frac{\zeta'(L^e)}{w(h)} - u'(c_e^*) = \frac{\sum_{p < p^*} \lambda(p)p [\zeta'(L^e)/w(h) - u'(c^e(p))]}{p^* + p^* \sum_{p < p^*} \lambda(p)} \geq 0. \quad (33)$$

The marginal tax rate on labor income $\Phi(p^*)$ is proportional to the (implicit) marginal tax rate on agents following all binding strategies, which, as we have proved, contemplate a lower level of effort than p^* . Naturally, no agent actually follows a different strategy. These are off-equilibrium choices which must be well understood for us to access the optimal marginal tax rate on labor income.

What (33) shows is that the marginal tax rate is non-positive. However if we add the following assumption, we may guarantee that the inequality in (33) is strict, which means that the marginal tax rate on labor income is negative.

Assumption B: *There exists an (arbitrarily small) $p > 0$ such that $\varphi(p) = 0$.*

This assumption guarantees that even if one does not make any effort to find a job there is a positive probability that she will find a job at the legal markets.

Proposition 6. *Under Assumption B, the marginal tax rate on labor income is negative at the optimum.*

Notice that Assumption B is sufficient, not necessary, for proposition 6. What is interesting about proposition 6 is the fact that this result was not present in the case where savings were observed. Nor is it part of any optimal unemployment insurance scheme derived in the literature.

Conditional on one's participating in the legal markets her labor supply, L^e is independent of her labor market attitude, p . This explains the zero marginal taxes prescription in the framework of section 5. What is new here is the fact that differences in savings affect the propensity to work conditional on one's being in the legal markets. Because off-equilibrium agents save too much, at the undistorted L^e , their marginal disutility of work exceeds their marginal utility of consumption. Subsidizing work produces first order welfare losses on off-equilibrium agents thus relaxing incentive constraints.

An important caveat is that this result may not be robust to relaxing the two period formulation, *if* one considers the possibility of multiple unemployment spells. However, the result should still be valid in a multi-period setting under the assumption, adopted in most of the literature, that once a worker gets a job she remains employed for the rest of her life.²¹

²¹E.g. Shavel and Weiss (1979) and Hopenhayn and Nicolini (1997). Wang and Williamson (1996) is a noteworthy exception.

An issue we have not investigated here is the possibility of hidden human capital investment. When we speak of human capital what we have in mind is more than simply years of schooling, which is what governments usually have some control over. This being the case, the use of the sophisticated tax instruments required to induce the optimal choices of h (see discussion in section 5.1) is not feasible. Government intervention is still possible through subsidies to direct costs of schooling, which we have not allowed for here. The intuition from the previous results are still valid and we do believe that subsidizing schooling will prove to be optimal.

8 Conclusion

In a two period model that subsumes life-long choices, we investigate the interaction between UI programs and educational policies. Agents' employment status is assumed to be affected by labor market attitude, which in its own turn is dependent on the relative cost of being unemployed. Education is important in this world not only because it raises expected income but also because it affects the opportunity cost of unemployment. It is this latter effect that plays the most prominent role in our model.

Our main result is that unemployment insurance and educational policies are complementary, i.e., in order to alleviate the moral hazard which is inherent to UI programs it is always optimal for the government to distort agents choices toward over-investment in human capital. Another important finding is that, despite the fact that there remains some consumption risk at the optimum — due to its being a constrained optimum in which moral hazard plays a role — the expected benefit of education is equal to its expected cost: a form of production efficiency result in our setup.

This latter result, however, depends on the government being able to make optimal inter-temporal transfers. The problem is that, as in Rogerson (1985), optimal policies require the expected marginal utility of consumption in the second period to be higher than marginal utility of consumption in the first period. This raises all types of questions about observability of savings and the potential non-convexities that arise when observability is not assumed.

We deal with non-observable savings and show that encouragement of education is robust to this modification in our main setup. Marginal income taxes, however, depend on whether savings are observed or whether they are not.

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A Appendix

A.1 Results from Section 6.

We start with the relaxed program for which constraint (23) is replaced by

$$\begin{aligned} u(c) - \zeta(h(p) + Y) + pV^e(h(p)) + (1-p)V^u(\omega) - \varphi(p) &\geq \\ u(c) - \zeta(h(\tilde{p}) + Y) + \tilde{p}V^e(h(\tilde{p})) + (1-\tilde{p})V^u(\omega) - \varphi(\tilde{p}) &\end{aligned} \quad (34)$$

for all $\tilde{p} \leq p$

$$Y - c + p(Y^e - c^e) - (1-p)\omega \geq 0.$$

Lemma 6 shows that the solution to this program solves the government program.

Lemma 3. *At all strategies that imply $\hat{p} < p$ agents accumulate less human capital than at strategy p .*

Proof. Note that, for given p, Y, Y^e the program is convex in h . The first order condition with respect to h is

$$-\zeta'(h + Y) + p\zeta'\left(\frac{Y^e}{w(h)}\right) \frac{w'(h)}{w(h)^2} = 0.$$

Hence, if $h(p)$ solves (22) and $h(\hat{p})$ solves the analogous expression with \hat{p} substituting for p , convexity of $\zeta(\cdot)$ implies $h(p) \leq h(\hat{p}) \iff p \leq \hat{p}$. *Q.E.D.*

Lemma 4. *The resource constraint binds at the optimum*

Proof. If there are any idle resources, increasing consumption in the first period increases welfare and does not affect incentives. *Q.E.D.*

Note that, despite the simplicity of direct mechanism argument, one should note that with regards to the tax system that implements it, tax rates on labor income and savings must be altered to guarantee that choosing this new allocation is rational.

Lemma 5. *At the optimum, at least one incentive constraint (34) binds.*

Proof. Suppose not. Then concavity of u imposes $c = c^e = c^u$. In this case it is optimal for the agent to choose $p = p_0$ and $h(p_0) = 0$. Assumption A, suitably adapted to this discrete case, guarantees that this cannot be optimal. *Q.E.D.*

Lemma 6. *The solution to the relaxed program maximizes the main program.*

Proof. Assume that, at the optimum, there is $\hat{p} > p$ such that

$$\begin{aligned} & -\zeta(h(\hat{p}) + Y) + \hat{p}V^e(h(\hat{p})) + (1 - \hat{p})V^u(\omega) - \varphi(\hat{p}) \\ \geq & -\zeta(h(p) + Y) + pV^e(h(p)) + (1 - p)V^u(\omega) - \varphi(p) \end{aligned}$$

In this case, note that

$$Y - c + \hat{p}(Y^e - c^e) - (1 - \hat{p})\omega \geq Y - c + p(Y^e - c^e) - (1 - p)\omega$$

Now, replace p by \hat{p} and there will be idle resources. From lemma 4 it is possible to increase welfare, thus contradicting the optimality of p . *Q.E.D.*

Proof of Proposition 3. Associated with the relaxed program is the Lagrangian,

$$\begin{aligned} \mathcal{L} = & u(c) - \zeta(h(p) + Y) + pV^e(h(p)) + (1 - p)V^u(\omega) - \varphi(p) + \\ & \sum_{\tilde{p} < p} \lambda(\tilde{p}) \{ [-\zeta(h(p) + Y) + pV^e(h(p)) + (1 - p)V^u(\omega) - \varphi(p)] - \\ & \quad [-\zeta(h(\tilde{p}) + Y) + \tilde{p}V^e(h(\tilde{p})) + (1 - \tilde{p})V^u(\omega) - \varphi(\tilde{p})] \} + \\ & \mu \{ Y - c + p(Y^e - c^e) - (1 - p)\omega \} \end{aligned} \quad (35)$$

Differentiating the Lagrangian and manipulating the first order conditions we characterize optimal policies. The first order condition of the Lagrangian with respect to Y is

$$-\zeta'(h(p) + Y) - \sum_{\tilde{p} < p} \lambda(\tilde{p}) [\zeta'(h(p) + Y) - \zeta'(h(\tilde{p}) + Y)] = -\mu,$$

which combined with the first order condition with respect to c ,

$$u'(c) = \mu,$$

yields

$$\underbrace{1 - \frac{\zeta'(h(p) + Y)}{u'(c)}}_{\tau} = \sum_{\tilde{p} < p} \lambda(\tilde{p}) \left[\frac{\zeta'(h(p) + Y)}{u'(c)} - \frac{\zeta'(h(\tilde{p}) + Y)}{u'(c)} \right] > 0, \quad (36)$$

where the sign of the left hand side of (36) is immediate from lemma 3. Q.E.D.

Proof of Proposition 4. The first order conditions with respect to Y^e and c^e are, respectively,

$$\begin{aligned} \mu &= \zeta' \left(\frac{Y^e}{w(h(p))} \right) \frac{w'(h(p))}{w(h(p))^2} \\ &\quad + \sum_{\tilde{p} < p} \lambda(\tilde{p}) \left[\zeta' \left(\frac{Y^e}{w(h(p))} \right) \frac{w'(h(p))}{w(h(p))^2} - \frac{\tilde{p}}{p} \zeta' \left(\frac{Y^e}{w(h(\tilde{p}))} \right) \frac{w'(h(\tilde{p}))}{w(h(\tilde{p}))^2} \right] \end{aligned}$$

and

$$\mu = u'(c^e) \left[1 + \sum_{\tilde{p} < p} \lambda(\tilde{p}) \left(1 - \frac{\tilde{p}}{p} \right) \right].$$

Combining the two,

$$\begin{aligned} 1 - \frac{\zeta'(Y^e/w(h(p)))}{w(h(p)) u'(c^e)} &= - \sum_{\tilde{p} < p} \lambda(\tilde{p}) \left[1 - \frac{\zeta'(Y^e/w(h(p)))}{u'(c^e)} \frac{w'(h(p))}{w(h(p))^2} - \right. \\ &\quad \left. \frac{\tilde{p}}{p} \left(1 - \frac{\zeta'(Y^e/w(h(\tilde{p})))}{u'(c^e)} \frac{w'(h(\tilde{p}))}{w(h(\tilde{p}))^2} \right) \right] \end{aligned}$$

obtains. The term in parenthesis in the left hand side is always positive since, from lemma 3,

$$1 - \frac{\zeta'(Y^e/w(h(p)))}{u'(c^e)} \frac{w'(h(p))}{w(h(p))^2} > 1 - \frac{\zeta'(Y^e/w(h(\tilde{p})))}{u'(c^e)} \frac{w'(h(\tilde{p}))}{w(h(\tilde{p}))^2},$$

and $\tilde{p} < p$. Q.E.D.

A.2 Results from Section 7.

The next three lemmas guarantee that labor supply responses are not strong enough to overcome the direct effect of transfers and savings on consumption.

Lemma 7. *At a fixed s , c^e is decreasing and c^u is increasing in transfers.*

Proof. For the first part we just note that $c^e = Y^e + s - \omega$. Since Y^e is chosen by the government, we have $dc^e/d\omega = -1 < 0$. For the second, assume for ease of exposition that $\delta = 0$ and note that $c^u = Y^u + s + \alpha\omega$ where $\alpha = p/(1-p)$ which means that $dc^u/d\omega = \alpha + dY^u/d\omega$. Then, $u'(Y^u + s + \alpha\omega) - \zeta'(Y^u) = 0$, which implies

$$\frac{dY^u}{d\omega} = -\alpha \frac{u''(c^u)}{u''(c^u) - \zeta''(Y^u)} \text{ and } \frac{dc^u}{d\omega} = \alpha \left[1 - \frac{u''(c^u)}{u''(c^u) - \zeta''(Y^u)} \right] > 0.$$

Q.E.D.

Lemma 8. c_0 is decreasing and c_u and c_e are increasing in s .

Proof. The proof follows the steps of lemma 7.

Q.E.D.

Lemma 9. $c_e^* \geq c_u^*$ in any relaxed program.

Proof. We will consider the relaxed program and we will prove the lemma by showing that, if $c_e^* < c_u^*$, a redistribution of income from the unemployment state to the employment increases welfare and is incentive compatible.

Define $(c_0^*, c_e^*, c_u^*, Y_u^*, Y_0^*, s^*) \equiv$

$$\arg \max \left\{ \begin{array}{l} u(c_0) - \zeta(Y_0) + p^* \left[u(c^e) - \zeta\left(\frac{Y^{e*}}{w(h)}\right) \right] + (1 - p^*)[u(c^u) - \zeta(Y^u)] - \varphi(p^*) \\ \text{s.t. } c^e = y^{e*} + s, c^u = Y^u + s + \omega, \text{ and, } c_0 = Y_0^* - s + \delta, \end{array} \right. \quad (37)$$

and $(c_0(p), c_e(p), c_u(p), Y_0(p), Y_u(p), s(p)) \equiv$

$$\arg \max \left\{ \begin{array}{l} u(c_0) - \zeta(Y_0) + p \left[u(c^e) - \zeta\left(\frac{Y^{e*}}{w(h)}\right) \right] + (1 - p)[u(c^u) - \zeta(Y^u)] - \varphi(p) \\ \text{s.t. } c^e = y^{e*} + s, c^u = Y^u + s + \omega, \text{ and, } c_0 = Y_0^* - s + \delta \end{array} \right. \quad (38)$$

Since p^* maximizes the relaxed program, we should have, for all $p < p^*$,

$$\begin{aligned} & u(c_0^*) - \zeta(Y_0^*) + p^* \left[u(c_e^*) - \zeta\left(\frac{Y^{e*}}{w(h)}\right) \right] \\ & + (1 - p^*)[u(c_u^*) - \zeta(Y_u^*)] - \varphi(p^*) \\ & \geq u(c_0(p)) - \zeta(Y_0(p)) + p \left[u(c^e(p)) - \zeta\left(\frac{Y^e}{w(h)}\right) \right] + \\ & (1 - p)[u(c^u(p)) - \zeta(Y^u(p))] - \varphi(p) \end{aligned} \quad (39)$$

Now, the fact that choices in (38) are optimal when the probability is p guarantees that

$$\begin{aligned} & u(c_0^*) - \zeta(Y_0^*) + p \left[u(c_e^*) - \zeta\left(\frac{Y^{e*}}{w(h)}\right) \right] + \\ & (1 - p)[u(c_u^*) - \zeta(Y_u^*)] - \varphi(p) \\ & \leq u(c_0(p)) - \zeta(Y_0(p)) + p \left[u(c^e(p)) - \zeta\left(\frac{Y^e}{w(h)}\right) \right] + \\ & (1 - p)[u(c^u(p)) - \zeta(Y^u(p))] - \varphi(p) \end{aligned} \quad (40)$$

From (39) and (40) we have

$$\begin{aligned} & u(c_0^*) - \zeta(Y_0^*) + p^* \left[u(c_e^*) - \zeta\left(\frac{Y^{e*}}{w(h)}\right) \right] + (1 - p^*)[u(c_u^*) - \zeta(Y_u^*)] - \varphi(p^*) \geq \\ & u(c_0^*) - \zeta(Y_0^*) + p \left[u(c_e^*) - \zeta\left(\frac{Y^{e*}}{w(h)}\right) \right] + (1 - p)[u(c_u^*) - \zeta(Y_u^*)] - \varphi(p), \end{aligned}$$

which implies that

$$\Delta p \left[\zeta(Y_u^*) - \zeta\left(\frac{Y^{e*}}{w(h)}\right) \right] \geq \Delta p[u(c_u^*) - u(c_e^*)] + \varphi(p^*) - \varphi(p), \quad (41)$$

where $\Delta p = p^* - p$. Observe that the deviation strategies generally contemplate different choices of s and Y^u , as long as $c_e^* \neq c_u^*$. Notice, however, that, if $c_e^* = c_u^*$, we have $s(p) = s^*$ and $Y^u(p) = Y_u^*$. Assume that $c_e^* < c_u^*$. We, now, distribute income from the unemployment state to the employment until we have $c_e^* = c_u^*$. This is feasible according to lemma 7. Denoting \hat{Y}^u the choice made by the truth-telling strategy after the reform, we have $\hat{Y}^u > Y_u^*$, (see the proof of lemma 7). We shall prove that the reform does not violate incentive compatibility, i.e.,

$$\begin{aligned} & u(\hat{c}_0^*) - \zeta(\hat{Y}_0^*) + p^* \left[u(\hat{c}_e^*) - \zeta\left(\frac{Y^e}{w(h)}\right) \right] \\ & + (1 - p^*)[u(\hat{c}_u^*) - \zeta(\hat{Y}_u^*)] - \varphi(p^*) \\ & \geq u(\hat{c}_0(p)) - \zeta(\hat{Y}_0(p)) + p \left[u(\hat{c}^e(p)) - \zeta\left(\frac{Y^e}{w(h)}\right) \right] \\ & + (1 - p)[u(\hat{c}^u(p)) - \zeta(\hat{Y}^u(p))] - \varphi(p) \end{aligned} \quad (42)$$

Because $\hat{c}_0^* = \hat{c}_0(p)$, $\hat{Y}_0^* = \hat{Y}_0(p)$, $\hat{c}_e^* = \hat{c}_0^e(p)$, $\hat{c}^u(p) = \hat{c}_u^*$ and $\hat{Y}_u^* = \hat{Y}^u(p)$, since $c_e^* = c_u^*$, after the reform, inequality (42) collapses to

$$\Delta p \left[\zeta(\hat{Y}^{u*}) - \zeta\left(\frac{Y^e}{w(h)}\right) \right] \geq \varphi(p^*) - \varphi(p) \quad (43)$$

Now, the right hand side of (43) minus the right hand side of (41) is $\Delta p \int_{Y_u^*}^{\hat{Y}^{u*}} \zeta'(Y) dY > 0$ and the left hand side of (41) minus the left hand side of (43) is $\Delta p[u(c_u^*) - u(c_e^*)] > 0$. Therefore, we conclude that the reform is incentive-compatible and increases welfare, since the utility is strictly concave. Q.E.D.

We are now in a position to prove the first two lemmas in Section 7.

Proof of lemma 1. Let s^* and $s(p)$ be as defined in (37) and (38), respectively, for

$p < p^*$. Assume that $s(p) < s^*$. (In which case $s^* > 0$). From lemma 8, this implies $c_0(s) > c_0^*$ which, in turn gives $u'(c_0(p)) < u'(c_0^*)$. Now, $u'(c_0(p)) \geq pu'(c_e(p)) + (1-p)u'(c_u(p)) > pu'(c_e^*) + (1-p)u'(c_0^*) > p^*u'(c_e^*) + (1-p^*)u'(c_0^*) = u'(c_0^*)$, where we invoked lemma 8, once again to derive the first inequality. This is, however, a contradiction. Q.E.D.

Proof of lemma 2. We have from lemma 1 that $s^* \leq s(p)$ whenever $p < p^*$. But, then, by an argument identical to the one used in the proof of lemma 7, one can easily show the result. Q.E.D.

The next lemma proves that resource constraints are binding at the optimum. This is not a trivial issue in a dynamic agency problem so a careful demonstration is needed.

Lemma 10. *The resource constraint multiplier, μ , for the relaxed program, (30), is positive.*

Proof. We first show that $Y^e = 0$ cannot be part of the solution to (30). First note that when $Y^e = 0$ the government only intervenes in the equilibrium of this economy by transferring resources across time. It is clear that at $Y^e = 0$ the welfare is lower than in the competitive equilibrium, when the government plays the same role of transferring resources. Hence, Y^e cannot solve the problem.

Consider, then, the case where solution with respect to Y^e is interior. Take the first order condition with respect to Y^e ,

$$p^* \mu - \frac{1}{w(h)} \zeta' \left(\frac{Y^e}{w(h)} \right) \left[p^* - \sum_{p < p^*} \lambda(p) (p^* - p) \right] = 0$$

which, then, implies

$$\mu = \frac{1}{w(h)} \zeta' \left(\frac{Y^e}{w(h)} \right) \left[1 - \sum_{p < p^*} \lambda(p) (1 - p/p^*) \right] > 0.$$

Q.E.D.

The next two propositions contain the main results regarding the usefulness of our approach. They guarantee that the solution to the relaxed program is the solution to the government's program, and that at least one IC constraint is binding at the optimum.

Proposition 7. *No constraint relative to a strategy that contemplates $p > p^*$ is binding at the optimum.*

Proof. First solve the relaxed problem and find the value p^* that solves (30). If there is no deviation strategy in which the agent chooses a higher level of effort and that yields at least the same utility level as the one associated with p^* , then, we have proved our proposition. For every $p \in \{0, p^1, \dots, 1\}$ define

$$W(p) \equiv \max_{(\hat{s}, \hat{Y})} \left\{ V(\hat{s}, \hat{Y}, \omega, h) + pV^e(\hat{s}, h, y^e, Y^e) + (1-p)V^u(\hat{Y}, \hat{s}, \delta) - \varphi(p) \right\},$$

and let $\bar{W} \equiv \max_p W(p)$ and $\bar{p} \equiv \max \{p; W(p) = \bar{W}\}$. Next, observe that $\bar{p}(Y^{e*} - w^{e*}) - (1 - \bar{p})w^{u*} - \omega^* > p^*(Y^{e*} - w^{e*}) - (1 - p^*)w^{u*} - \omega^* \geq 0$. Hence, resources are idle, which implies, from lemma 10, that $\{w^{e*}, h^*, w^{u*}, Y^{e*}\}$ is not a solution to the \bar{p} -relaxed program. This contradicts the assumption that p^* belongs to the solution of (30). Q.E.D.

Proposition 8. *At least one incentive compatibility constraint binds at the optimum.*

Proof. Assume the contrary. It is clear that the government must provide full insurance. Hence, it is obvious that no agent would have any incentive to choose a positive effort. Therefore, this policy would not be feasible. Q.E.D.

We may, now strengthen the result in lemma 9.

Lemma 11. *At the optimum $c_e^* > c_u^*$.*

Proof. Recall the Lagrangian for the government's problem,

$$\begin{aligned} \mathcal{L} \equiv & V(\delta, h, y^e, Y^e, \omega, \hat{p}) + \mu[\hat{p}(Y^e - y^e) - (1 - \hat{p})\omega - w] \\ & + \sum_{p < \hat{p}} \lambda(p)[V(\delta, h, y^e, Y^e, \omega, \hat{p}) - V(\delta, h, y^e, Y^e, \omega, p)]. \end{aligned}$$

In this case,

$$\left. \frac{\partial \mathcal{L}}{\partial \omega} \right|_{c_e^* = c_u^*} = \frac{\partial V_0}{\partial \omega} - \mu + \sum_{p < p^*} \lambda(p) \left[\frac{\partial V(p^*)}{\partial \omega} - \frac{\partial V(p)}{\partial \omega} \right] = -\mu < 0.$$

Q.E.D.

We now show that there is a deviating strategy that binds at the optimum and for which savings are greater than at the equilibrium choices.

Lemma 12. *At the optimum, there is $p < p^*$ such that $s(p) > s(p^*) \geq 0$ and $\lambda(p) > 0$.*

Proof. First, the existence of $\lambda(p) > 0$ is due proposition 8 and the Kuhn-Tucker theorem. Now, if $u'(c_0^*) = \mathbb{E}u'(c_1^*)$, a slight change in the proof of lemma 9 shows that $s(p) > s(p^*)$. So, let us suppose that $u'(c_0^*) > \mathbb{E}u'(c_1^*)$ (consequently, $s(p^*) = 0$) and that $s(p) = 0$ for all $p < p^*$ such that $\lambda(p) > 0$. In this case, $u'(c_0^*) \geq u'(c_0(p))$, from lemma 2. The first order condition for the planners' problem gives (if $s(p) = s(p^*) = 0$) $u'(c_u^*) = \mu - \sum_{p < p^*} \lambda(p)(p - p^*)u'(c_u^*)$, and $u'(c_e^*) = \mu - \sum_{p < p^*} \lambda(p)(p^* - p)u'(c_e^*)$. Therefore, from the strict concavity of $u(\cdot)$ we have that $c_u^* < c_e^*$. Next, lemma 1 and the fact that $s(p) = 0$ for all $p < p^*$ with $\lambda(p) > 0$, leads to $u'(c_0^*) = u'(c_0(p))$ for all $p < p^*$ such that $\lambda(p) > 0$. Moreover, from the fact that $c_u^* < c_e^*$, it is clear that $\mathbb{E}u'(c_0^*) < \mathbb{E}u'(c_0(p))$. We will now show that for $\varepsilon > 0$ sufficiently low, the policy $\{w^{e*} - \varepsilon, w^{u*} - \varepsilon, \omega^* + \varepsilon\}$ is welfare-improving and clearly does not violate the resource constraint. For ε sufficiently low, $\Delta W \approx$

$$\varepsilon \left\{ u'(c_0^*) - \mathbb{E}u'(c_0^*) + \sum_{p < p^*} \lambda(p)u'(c_0^*) - \mathbb{E}u'(c_0^*) - u'(c_0(p)) + \mathbb{E}u'(c_0(p)) \right\},$$

or

$$\Delta W \approx \varepsilon \left\{ u'(c_0^*) - \mathbb{E}u'(c_0^*) + \sum_{p < p^*} \lambda(p)\mathbb{E}u'(c_0(p)) - \mathbb{E}u'(c_0^*) \right\} > 0,$$

which contradicts the optimality of the policy.

Q.E.D.

Proof of Proposition 6. We re-write (33) more compactly as

$$\Phi(p^*) = \kappa(p^*) \sum_{p < p^*} \pi(p) \Phi(p) \geq 0, \quad (44)$$

where

$$\kappa(p^*) \equiv \frac{\sum_{p < p^*} \lambda(p)pu'(c^e(p))}{\left(p^* + p^* \sum_{p < p^*} \lambda(p)\right)u'(c_e^*)}, \quad \pi(p) \equiv \frac{\lambda(p)pu'(c^e(p))}{\sum_{p < p^*} \lambda(p)pu'(c^e(p))},$$

and

$$\Phi(p) \equiv \frac{\zeta'(L^e)}{w(h)u'(c^e(p))} - 1.$$

From lemma 12, there is $p < p^*$ such that $s(p) > s(p^*) \geq 0$ and $\lambda(p) > 0$. From the Spence-Mirrlees condition, it is clear that $s(p) > s(p^*) \Rightarrow \Phi(p^*) < \Phi(p)$. Now, if the left hand side of (44) is zero, the right hand side is negative. Therefore, the marginal tax rate can not be zero.

Suppose, however, that $\Phi(p^*) < 0$. In this case,

$$0 < \frac{\sum_{p < p^*} \lambda(p)p}{p^* \left(1 + \sum_{p < p^*} \lambda(p)\right)} < 1.$$

The right hand side is less than the left hand side, hence this can not be the case.

Q.E.D.